# Routing Strategies for IP Networks 

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#### Abstract

This work addresses the problem of static routing complexity and performance for best effort traffic in a data network and more specifically an Internet network running an IGP (Interior Gateway protocol), and MPLS if necessary. We first give a short presentation of the various routing strategies (single-path and multi-path) and their possible realization in an IP intra domain network. We then briefly introduce the problem of the performance measurement of a routing pattern. We also define the complexity of a routing pattern as the number of MPLS tunnels needed for its realization. We show how the number of MPLS tunnels that are needed to enhance an IGP routing strategy can be minimized. We compare different routing strategies in IP networks from the two points of view: complexity and performance. We then propose two off-line Traffic Engineering methodologies for IP intra-domain network: the first one is based on an IGP/MPLS architecture; the second one is based only on the IGP routing using an optimized load balancing scheme. The algorithms used to compute the IGP metric and to optimize the routing patterns are also briefly described.


## Keywords

Internet Networks, Routing Strategies, Routing Performance, Administrative Metric, Traffic Engineering, IGP, MPLS, Algorithms.

## 1 Introduction

Traffic routing within a telecommunication network defines how the traffic matrix is mapped on the network topology. Routing mechanisms are thus identified as an essential feature in the control of the network performance [Awduche_1]. The routing mechanisms involved allow to assign the network capacities, more or less efficiently, to the demands. The routing choice has a direct impact on the existence and location of congestion within the network. A high level of congestion may decrease the grade of service (call blocking, increased delays, packet losses, etc).

Routing mechanisms within an IP network may induce some restrictions on the path choice related to the path selection algorithm. The problem occurs more specifically in the case of a IP networks running an IGP (Interior Gateway Protocol) routing protocol. In this case, the routes derive from very simple routing algorithms (shortest path calculations) which offer only limited control over the routing paths. This often leads to a sub-optimal utilization of the network resources. Today several new mechanisms are proposed to increase the routing control and to optimize the network performance, and among them MPLS. However such mechanisms also introduce some complexity in the network management. We try to analyze the compromise between routing performance and complexity. We propose two off-line Traffic Engineering methodologies: the first one is based on an IGP/MPLS architecture; the second one is based only on the IGP routing using an optimized load balancing scheme.

## 2 Organization of the paper

We introduce various (static) routing strategies (single-path and multi-path routing strategies) and describe how they can be specifically realized in an IP intra domain network (Section 3).

We then present some of the routing performance criteria that can be optimized (Section 4). We also introduce the complexity of an IP routing strategy as the number of MPLS tunnels needed.

The performance and complexity of various IP routing strategies are then compared according to the most heavily loaded link criterion (Section 5).

Some classes of efficient routings strategies are selected from these comparisons and two off-line Traffic Engineering methodologies are derived (Section 6).

Section 7 is devoted to the algorithms used in the context of performance optimization.

## 3 Some static routing patterns

We first need the following definitions:
Network topology: we assume that we can represent the network topology as a simple non oriented graph that is represented by its nodes and edges. Multiple parallel links are represented by a unique edge between the nodes.
$\rightarrow \quad$ Note that in MPLS Traffic Engineering although $n$ parallel links can be announced as a single bundled link [Kompella], in order to use all links capacity, $n$ parallel LSPs must be established (unless a solution based on LSP hierarchy is used [Kompella_2]). For IGP routing see ECMP below.

Routing pattern: for a given network topology, we define a routing pattern as a set of (possibly multiple) directed routes between pairs of nodes in the network. If there is at least one route in each direction between each pair of nodes, the routing pattern is fully meshed.

Various static routing patterns are introduced here with their possible realization in an IP intradomain network. We also focus on some specific IP routing strategies based on the modification of the IGP routing with ER-LSP (Explicit Routed Label Switched Path) created with MPLS.

In the sequel the terms ER-LSP, tunnel, and MPLS tunnel are indifferently used.

### 3.1 Single-path routing patterns

In a single-path routing pattern there is at most one route between each pair of nodes. We can distinguish symmetric single-path routing patterns if the paths between $A$ and $B$ and $B$ and $A$ use the same edges for all pair of nodes (A,B). Single-path routing patterns may be divided in the following interesting sub-classes:

- Shortest path routings patterns: if there exists a metric (a set of pairs of values, one for each direction, on the edges of the network) such that all paths of the routing pattern are a shortest path between the end-points according to that metric. A special case is when all shortest paths are also unique (unique shortest path).
$\rightarrow \quad$ Classical intra domain routing protocols (OSPF, IS-IS) are based on such shortest path calculations. Administrative metric values are related to the system interfaces: between two routers a different metric value can be affected to each interface of a same link. Resulting routing patterns can thus be symmetric or not.


Figure 1: The sub-optimality condition

- Routing patterns satisfying a sub-optimality (SO) property: two given paths having two points in common satisfy the sub-optimality condition if they share the same sub-path between these two points (Figure 1). Note that this sub-optimality condition excludes traffic load balancing and load distribution which aims to divide at an intermediate node the traffic toward the same destination on several distinct paths. Note also that routing patterns satisfying the SO condition are necessarily symmetric. Routing patterns based on unique shortest paths satisfy the sub-optimality condition when the metric values are the same on the two interfaces of a link. The contrary is false [BenAmeur\&Gourdin_1].
- Destination-based single-path routing: any packet is forwarded through the network using the destination address. Obviously, shortest path routing and sub-optimal routing are also based on destination. However, this class of routing patterns is larger. In fact, this is equivalent to establishing a spanning tree for each destination. The destination trees can be completely independent.
- General single-path routing patterns without constraints: the whole traffic demand between an origin-destination pair is routed through a single path without any additional constraint.
$\rightarrow \quad$ In an IP network running a classical IGP routing protocol, only shortest path routing patterns can be realized. Other single-path routing patterns can be realized with the explicit routing functionality enabled by MPLS (strict ER-LSP). As an ER-LSP is always unidirectional, symmetric or directional routing patterns can be realized. When the routing pattern is fully meshed, the total number of ER-LSP to create is equal to $n^{*}(n-1)$ where $n$ is the number of nodes.

In the sequel, for the sake of simplicity of the study, we focus our attention on symmetric single-path routing patterns only. Note that for operational reasons this property is often required by network operators. One reason is to limit the complexity of management of the network. Another reason is to prevent to have a routing path up in one direction while the return routing path is down due to a link failure. With symmetric routing patterns, routing paths in both directions are simultaneously up or down in case of link failure.

### 3.2 Multi-path routing patterns

In a multi-path routing pattern, traffic between two nodes can be forwarded among several distinct paths.

In IP networks, load sharing can be achieved at an intermediate node in multiple ways: on a packet per packet basis, or with a hashing function evaluated from the information read in the packet header, etc. A hashing function based on the origin and destination can achieve sufficient granularity in a core network.
$\rightarrow \quad$ An IGP routing protocol can provide multiple equal cost paths between which load sharing can be implemented. Because there is no information in current IGP routing protocols about traffic loading on distant links, techniques have been utilized to divide traffic somewhat evenly among the available paths. Those techniques are referred to as Equal Cost MultiPath (ECMP). A classical utilization of ECMP is to assign the same metric to parallel links between two routers so that all those links will be used to forward traffic. This is thus equivalent to single-path routing in our topology model where we consider multiple parallel links as a unique (aggregated) link. Another technique, Optimized MultiPath (OMP) [OSPF-OMP], tries to adjust the load balancing parameters at each node in function of the network load. This requires significant change to the IGP because dynamic information is needed in each router about link loads in the network. This proposition was never implemented;


Figure 2: General ECMP
$\rightarrow \quad$ General ECMP: instead of splitting the traffic evenly between the shortest paths, we can split it in any arbitrary way. In fact, it is very easy to see that when no particular routing constraints are added (number of hops for example), the link loads of any multi-path routing pattern can be
reproduced by a routing strategy where forwarding is based only on destination. That is to say, a node B who have to route a packet to A, will randomly choose a path (an interface) using only the destination address. In other terms, if a certain proportion of the traffic demands from C to A and from D to A, use B as an intermediate node, then this traffic will be split in the same way between B and A whatever the origin (C or D) (Figure 2). We will show in Section 7 how a multi-path routing can be transformed into a shortest path routing.
$\rightarrow \quad$ With MPLS, several tunnels can be opened between a pair of nodes and traffic can be arbitrarily shared among them;

### 3.3 Specific routing patterns in IP networks

The realization of the routing patterns mentioned above is based either on the IGP routing or on administratively configured TE tunnels. Both mechanisms can be integrated : the IGP routing can be modified to take into account TE tunnels. Three different models can be identified: in the first two models, only the path selection process of the IGP in a node is modified taking into account the TE tunnels originating at this node, in the third model TE tunnels are advertised by the IGP protocol.

- «Basic IGP Shortcut" : if a packet arrives in a router where originates a tunnel with remote extremity the destination of the packet, then the packet is forwarded to the destination. Otherwise the packet follows the classical IGP routing;
- «IGP Shortcut» : in this model proposed at the IETF [Smit], the shortest path calculation in the routers remains unchanged but the determination of the next hop is modified in the following way: if a tunnel originates in the router with its extremity belonging to the shortest path, then the packet will be forwarded in this tunnel;
- "Advertise tunnels into the IGP": in this model implemented by some manufacturers, tunnels are advertised in the IGP and used in the shortest path calculations as virtual interfaces.

Depending on implementation details and in particular on the tunnels metric assignment, many different options are possible in the path selection process. They give more flexibility to the current IGP routing protocols: the resulting routing patterns will not necessarily be shortest paths, nor satisfy the SO condition nor even be destination based.

## 4 Routing performance criteria for best effort IP traffic

We consider static routing patterns and best effort traffic controlled by TCP. The performance of routing patterns can be viewed from the user's point of view or from the network's point of view. This distinction is introduced in [Awduche_2] where traffic oriented performance and resource oriented performance objectives are defined:
$\rightarrow \quad$ Traffic oriented performance: the quality of service perceived by end users is mainly determined by the (random) duration of a document transfer (Web page, e-mail, FTP file, etc). Since the source traffic rates are reactive to the network load (TCP behavior), the quality of service will depend on the link loads across the path;
$\rightarrow \quad$ Resource oriented performance: from the operator's point of view, the objective is to minimize resource utilization (link capacity). Another objective can be the robustness of the traffic repartition against traffic fluctuations. The first objective implies that a routing pattern must be found such that another routing cannot be found with a lower load on each link and with a strictly lower load for at least one link. Such a routing pattern is said to be non dominated The second objective can be partially addressed by looking for a routing pattern that minimizes the maximum
link load: such a routing pattern will be able to cope with the maximal traffic increase (with the assumption of a homogeneous traffic increase across all origin-destination demands)

For the sake of computational tractability, a simple performance criterium is required: it should be only related to the edge loads and capacities, but independent on the network topology and on the effectively used routing paths.

Notations:
We consider a network defined by its set of edges $L$ and a given static routing pattern. Let $C_{l}$ be the capacity of edge $l$ and $A_{l}$ be the average traffic load carried through this edge (this load effectively depends on the routes within the network). The average load of edge $l$ is defined as $\rho_{l}=A_{l} / C_{l}$. A routing pattern is said to be feasible if $\rho_{l} \leq 1$ for any edge.

Criteria based on the edge loads:
It seems natural to try to maximize a concave decreasing functions of the edge loads as for instance:

$$
\begin{equation*}
\frac{1}{1-\alpha} \sum_{l \in L}\left(1-\rho_{l}\right)^{1-\alpha}, \alpha \geq 0, \alpha \neq 1 \tag{1}
\end{equation*}
$$

This function was proposed and studied in [Mo\&Warland] and [Bonald\&Massoulié].
When $\alpha$ is close to 1 , the function (1) is equivalent to $\frac{\|L\|}{1-\alpha}+\sum_{l \in L} \log \left(1-\rho_{l}\right)$.
Therefore, for $\alpha=1$, criterion (1) can be extended and replaced by $\sum \log \left(1-\rho_{l}\right)$.
A routing is said to be optimal if it is able to carry the whole traffic flow minimizing criterion (1). An interpretation can be proposed for some values of $\alpha$ :
$\rightarrow \quad \alpha=0$ minimizes the average edge load. This is a simple criterion but we wouldn't recommend it because it is unable to differentiate two links with respective loads of $0 \%$ and $100 \%$ and two links $50 \%$ loaded (contrarily to the case $\alpha>0$, the function is not strictly concave);
$\rightarrow \quad \alpha=1$ maximizes $\sum \log \left(1-\rho_{l}\right)$, equivalently the geometric mean of $\left(1-\rho_{l}\right)$;
$\rightarrow \quad \alpha=2$ minimizes $\sum 1 /\left(1-\rho_{l}\right)$, equivalently the harmonic mean of $\left(1-\rho_{l}\right)$;
$\rightarrow \quad \alpha=\infty$ corresponds to a «min-max » criterion. One is successively interested in minimizing first the maximum load, then the second maximum load, and so on.

The higher the value of $\alpha$ is, the more attention is paid to the most heavily loaded edge.

## Criteria based on the edge residual capacities:

It is also possible to replace in (1) the edge load by the residual capacity $C_{l}\left(1-\rho_{l}\right)$ Objective functions of the following type can thus be considered:

$$
\begin{equation*}
\frac{1}{1-\alpha} \sum_{l \in L}\left(C_{l}\left(1-\rho_{l}\right)\right)^{1-\alpha}, \alpha \geq 0, \alpha \neq 1 \tag{2}
\end{equation*}
$$

Interpretations similar as for criteria (1) can be proposed. The higher the value of $\alpha$ is, the more attention is paid to the edge with the lowest residual capacity.

Note that a routing pattern achieving the optimum value for one of the criteria described above is a non-dominated solution.

The choice of a performance objective can be driven by the nature of the studied network, backbone or access network. Considering a backbone network, the customer bit rate is generally bounded by the access rate (or the rate of the Web server) which is small compared to the edge capacities. The traffic oriented performance criteria are thus less crucial than the network oriented performance ones. A criteria related to the most heavily loaded edge seems relevant in the case of static routing when the network is unable to adapt itself automatically to traffic fluctuations. The most heavily loaded edge criterion is one of the most often used criteria to evaluate the performance of backbone networks.

## 5 Comparison of static routing patterns

The following static routing strategies are compared (listed in a decreasing order of flexibility):
$\rightarrow \quad$ Multi-path symmetric routing;
$\rightarrow \quad$ Single-path symmetric routing;
$\rightarrow \quad$ Single-path symmetric routing with constraint of sub-optimality;
$\rightarrow \quad$ Unique symmetric shortest path routing;
$\rightarrow \quad$ Minimum hop (symmetric) routing.

In the sequel, it is implicit that all routing patterns considered are symmetric. We believe some of the results can be extended to asymmetric routing patterns but this is left for further study.

Remind that for any multi-path routing pattern, it is possible to find a destination based multipath routing scheme that achieves the same load links (see Section 3.2). This routing scheme can be implemented using a generalized ECMP technique.

Definitions :

1) for a given routing strategy and a given network topology, we call routing set of a routing strategy the set of all routing patterns that can be achieved with this routing strategy;
2) for a given routing strategy, a given network topology, and a given performance criterion, we call performance of a routing strategy the best performance of all routing patterns that can be achieved with this routing strategy.

We first define the notion of complexity of a routing strategy in an IP network. We then try to analyze the various routing patterns that can be achieved with the above routing strategies and the associated complexity. Finally we compare the performance of these routing strategies.

### 5.1 Complexity of the realization of a routing pattern in IP networks

The IGP routing protocol has some advantages: its simplicity, scalability, automated and distributed implementation. Moreover IGP routing has already proven its robustness and resilience. A disadvantage of using MPLS explicit routes is the administrative burden and potential for human induced errors from using this approach on a large scale [Michel\&al]. Network operators thus might want to minimize the total number of MPLS tunnels created in the network.

We define the complexity of a routing pattern as the number of tunnels that are needed for its realization in an IP network.

### 5.2 Scenarios

Several scenarios (topology and traffic matrix) have been selected in order to compare the different routing strategies. Some of them have been studied by C. Villamizar [Villamizar_1, Villamizar_2] in the evaluation of OMP approaches and the others have been extracted from real case world networks.

The scenarios used by Curtis Villamizar are available on his Web site along with the results of his simulations [Villamizar_2].

|  | Nodes | Edges | Mesh degree | Demands |
| :--- | :---: | :---: | :---: | :---: |
| OMP_10_29 | 10 | 29 | 5.8 | 45 |
| OMP_20_51 | 20 | 51 | 5.1 | 190 |
| OMP_50_101 | 50 | 101 | 4.0 | 1225 |

These scenarios are defined by a network topology (obtained by random generation) along with capacity on the edges and a traffic matrix. Edges are symmetric but may have a different capacity in each direction. The traffic matrix is oriented.

The two following scenarios extracted from real case networks have also been studied:

- Scenario FT_1 : 9 nodes 20 edges and 35 symmetric demands;
- Scenario FT_2 : 26 nodes 39 edges and 154 symmetric demands.


### 5.3 Comparison of routing sets: size and complexity

In what follows, we try to answer the following questions: what is the relative size of the routing sets of each routing strategy? What is the complexity of realization of the corresponding routing patterns in an IP network?

### 5.3.1 Shortest path routing

We first introduce some definitions:

1) A single path and a metric are compatible if the path is a unique shortest path according to the metric. A metric is compatible with a single-path routing pattern if all paths are compatible with the metric. In Section 7, we address the case where the constraint of uniqueness of a shortest path is relaxed;
2) A routing pattern is compatible if there exists a metric compatible with all paths in the routing pattern;
3) For a given single-path routing pattern the number of compatible paths is defined as the maximal number of paths of a compatible sub-routing pattern (a subset of paths of the routing pattern).
A first step in this routing strategy analysis is to measure the difficulty to find compatible metrics for a given routing pattern.. For different network topologies, we have randomly generated 100 fully meshed single-path routing patterns and 100 fully meshed single-path routing patterns satisfying the suboptimality condition. In each case a compatible metric has been searched using a linear programming method described in [Ben-Ameur\&Gourdin_1] and [Ben-Ameur\&Liau] (see Section7).

We remind that a routing pattern that is not satisfying the sub-optimality condition is never compatible [Ben-Ameur\&Gourdin_1].

|  | Number of compatible routing <br> patterns |  | Percentage of compatible paths <br> (in case of non compatible routing <br> pattern) |  |
| :--- | :---: | :---: | :---: | :---: |
|  | General single- <br> path routing <br> pattern | sub-optimality <br> compliant routing <br> pattern | General single- <br> path routing <br> pattern | sub-optimality <br> compliant routing <br> pattern |
| OMP_10_29 | $0 \%$ | $51 \%$ | $35 \%$ | $95 \%$ |
| OMP_20_51 | $0 \%$ | $2 \%$ | $29 \%$ | $88 \%$ |
| OMP_50_101 | $0 \%$ | $0 \%$ | $33 \%$ | $69 \%$ |

Although a limited number of topologies has been tested, we can draw the following trends from these results:
$\rightarrow \quad$ General single-path routing patterns: it seems difficult to find a compatible metric for general single-path routing patterns (not a single case in our tests). The routing set of single-path routing strategy is thus much larger than the routing set of the unique shortest path routing strategy. However it is possible to find a metric compatible with at least a significant sub-routing pattern: in average $30 \%$ of the paths whatever the size of the network;
$\rightarrow \quad$ Sub-optimality compliant routing patterns: in a significant number of cases it is possible to find a compatible metric. The size of the routing set of the sub-optimality compliant routing strategy seems to be very close to the size of the routing set of the unique shortest path routing strategy for (very) small networks (scenario OMP_10_29). As the size of the network increases (a few dozen of nodes), the size of the routing set of the sub-optimality compliant routing strategy seems to be again much bigger than the size of the routing set of the unique shortest path routing strategy (scenario OMP_20_51 and OMP_50_101). However the percentage of compatible routing paths is higher than for the general routing patterns (more than $70 \%$ ) although it seems to decrease with the size of the network.

These results depend on the studied topologies. For example, for a ring network the routing set of the sub-optimality compliant routing strategy is equal to the routing set of the unique shortest path routing strategy [Ben-Ameur\&Gourdin_1]. It is likely that the results depend on the degree of connectivity of the network. Other relevant topologies for IP networks are under study.

### 5.3.2 Single-path routing with metrics and tunnels

We have seen that a general single-path routing pattern is not often compatible. It is possible to realize such routing patterns in an IP network using strict explicit routing, for example by creating two ER-LPS per path, one in each direction. This requires $n *(n-1)$ MPLS tunnels in the network (if the routing pattern is fully meshed). The routing complexity is thus directly related to the number of demands.

However in the case of sub-optimality compliant routing patterns, it is often possible to find a metric compatible with a large percentage of the paths in the routing pattern. The question is now the following : is it possible to reproduce the remaining non-compatible paths with the IGP routing modified with a limited number of MPLS tunnels?

We consider the "IGP Shortcut" model of integration of the IGP routing with the MPLS tunnels (Section 3.3). For each remaining path not compatible with the metric, the two corresponding ER-LSP are created (one in each direction). The modified IGP routing will thus route the traffic along the correct paths for these routing paths not compatible with the metric. However those tunnels can modify the routes found by the modified IGP for the paths that are compatible with the metric.

It is easy to show the following result : if the initial routing pattern satisfies the sub-optimality condition, then the tunnels created as described above do not modify the IGP routing for the paths that were compatible with the metric. Thus, in the case where the routing pattern satisfies the sub-optimality condition, it can be realized by an IGP routing protocol modified by some tunnels. The number of pairs of tunnels (one in each direction) needed is equal to the number of paths in the routing pattern minus the number of compatible paths. However in some cases, it may be possible to create less tunnels because a pair of tunnels may modify more than one shortest path into the correct routing path (see Section 7.1.2).

### 5.3.3 Complexity of the routing patterns

We consider all routing patterns (including single-path and multi-path routing patterns) and their realization in IP networks. Some of them can be reproduced without any MPLS tunnels (i.e. using only the IGP routing), some others require the creation of a limited number of MPLS tunnels (IGP routing modified with some MPLS tunnels) and the last routing patterns require a large number of MPLS tunnels (in the order of the number of paths in the routing pattern).

Based on the results above, we can represent on Figure 3 a comparison of the complexity of different routing patterns.


Figure 3: Complexity of various routing patterns
We can see that a large number of routing patterns (much larger than the number of routing patterns that can be achieved with the IGP routing only) can be achieved with a "reasonable" complexity
(with a limited number of tunnels). The natural question that arises is the following: what level of performance can be achieved with each level of complexity?

### 5.4 Comparison of performance

The performance criteria considered in this Section concerns the network ability to support traffic increases. It is measured by the maximum edge load (Section 4).

### 5.4.1 Optimization

A different optimization problem has to be solved for each routing strategy. Some of them are NP-hard and cannot be solved exactly: in these cases a heuristic has been used. As a consequence, the comparison of the routing strategy performance may be affected by the accuracy of these heuristics. The routing optimization procedures we have used are described below:

- Multi-path routings: a linear programming (exact solution);
- Single-path routings: a heuristic (a branch and cut algorithm) based on linear programming which also provides an upper bound on the optimal solution [Geffard]. Only symmetric problems can be solved with this tool (consequently not the Villamizar scenarios ${ }^{1}$ );
- Single-path with constraint of sub-optimality: an exact solution (based on a linear programming) is under study [Ben-Ameur\&Gourdin_2].
- Unique shortest path: a simulated annealing heuristic [Ben-Ameur\&al].

More details about these optimization algorithms are given in Section 7.

### 5.4.2 Results

Table 1 summarizes the main results of our tests. In order to understand this table, note that :
$\rightarrow \quad$ A result marked with a * means that the solution value is optimal;
$\rightarrow \quad$ Results in bold characters were obtained by Villamizar and are directly reported from his Web site [Villamizar_2]: results for MPLS-OMP are used for the multi-path routing strategy and the single-path routing strategy (results are obtained with a simple greedy heuristic).

| Results | Multi-path | Single-path | Minimum <br> Hop Routing | Unique Shortest Path |
| :---: | :---: | :---: | :---: | :---: |
| OMP_10_29 | $\mathbf{0 . 6 1}$ (MPLS-OMP) | $\mathbf{0 . 8 3}$ | 1.15 | 0.85 |
| OMP_20_51 | $\mathbf{0 . 7 0}$ (MPLS-OMP) | $\mathbf{1}$ | 1.82 | 0.87 |
| OMP_50_101 | $\mathbf{0 . 6 9}$ (MPLS-OMP) | $\mathbf{0 . 8 8}$ | 1.60 | 0.82 |
| FT_9 | $0.78^{*}$ | $0.79^{*}$ | 2.93 | 0.80 |
| FT_26 | $0.64^{*}$ | $0.66^{\star}$ | 1.50 | 0.88 |

Table 1: Performance of different routing strategies
The following comments can be derived :

[^0]- Single-path versus multi-path routing: in the case of scenarios FT_9 and FT_26, the proposed solution is optimal and the performance of both routing strategies is very close. The result is quite different in the case of Villamizar scenarios. The single path constraint decreases the performance (about $30 \%$ ). Note that in the latter case the optimization heuristic used is very simple and we have no guaranty on the quality of the solution. Results seem to depend highly on the network topology and on the traffic matrix. Note that it is easy to build scenarios for which the performance of the single-path routing strategy is arbitrarily worse than the performance of the multi-path routing strategy (below is an example of a topology on which a single-path routing strategy will perform very badly compared to a multi path routing strategy because it is not possible to balance the traffic from O to D on the n parallel paths). However in an operational perspective, the worst case is not relevant, only the average case over realistic topologies;

- Shortest path routing versus minimum hop routing: the comparison between unique shortest path routing and minimum hop routing strategies illustrates the significant impact of a wise selection of the metric values. The choice of a default value (in the minimum hop routing strategy, the edge metric value is systematically set to one) may induce a very poor performance compared to the performance achievable with an optimized metric (in the studied scenarios, the relative performance drop from $25 \%$ up to $200 \%$ );
- Single-path routing versus unique shortest path routing:
$\rightarrow \quad$ Note that for the Villamizar scenarios, the performance achieved with unique shortest path strategy is sometimes better than with a less constrained single-path routing strategy. It only means that, in the case of single-path routing optimization, the heuristic is not accurate enough to reach a value close to the optimum. This may be of some importance, because such heuristics are quite often used, even in operational network configuration tools;
$\rightarrow \quad$ In the case of FT_9 and FT_26 scenarios, the optimal performance of the single-path routing strategy is found. For the smaller network (FT_9), the performance that can be achieved with the unique shortest path strategy is very close to this value. However for scenario FT_26, the best performance that can be achieved with the unique shortest path strategy is $30 \%$ worst than this value. Further tests are needed to investigate whether the gap increases with the size of the network (number of edges).


### 5.4.3 Performance improvement with MPLS tunnels

The size of the routing set for the unique shortest path routing strategy modified with a few MPLS tunnels if much larger than the size of the routing set for the unique shortest path routing strategy. A natural question then follows : is it possible to significantly improve the performance of unique shortest path routing by adding a few MPLS tunnels ?

We suppose that the IGP routing is modified by the MPLS tunnels according to the «IGP Shortcut» integration model (Section 3.3). For example, if we consider scenario OMP_10_29, the best performance achieved with the unique shortest path routing strategy is 0.85 . By looking at the routing paths, we note that 3 links have the maximum load of $85 \%$. We have identified 3 pairs of MPLS tunnels that lead to a modified routing pattern where the most heavily loaded link have a load of $77 \%$.

By creating a few MPLS tunnels, it is in some cases possible to realize a new routing pattern with a significantly improved performance. An important point to mention here is that the resulting routing pattern does not necessarily satisfy the sub-optimality condition. This means that it is possible to achieve some kind of load distribution where two demands may be routed on two paths with two nodes in common but using a distinct path between the 2 nodes (Figure 4).


Figure 4 : shortest path routing pattern modified by a TE tunnel thereby achieving load balancing
Finally, note that it is not clear which of the three different models of integration of the IGP routing with MPLS tunnels is the most interesting. The first one, however, may add more complexity because one tunnel can be used by only a limited number of demands.

## 6 « Off-line» Traffic Engineering methodologies

Based on the results of Section 5, we can propose off-line «Traffic Engineering » methodologies. The objective is to improve the performance of the network in terms of resource utilization. Two different methodologies are described: the first one using MPLS, the other one relying on the IGP routing only but and using a generalized ECMP technique. In both cases, a single class of (best effort) traffic is considered. It is also assumed that a representative end-to-end traffic matrix between the network nodes can be measured or estimated.

### 6.1 An MPLS based off-line Traffic Engineering methodology

The following assumptions are made:
$\rightarrow \quad$ MPLS is deployed in the network and it is possible to create explicitly routed MPLS tunnels (ER-LSP);
$\rightarrow \quad$ The IGP routing is modified to take into account the MPLS tunnels in the determination of the next hop according to the "IGP Shortcut" model (Section 3.3).


Figure 4: Off line Traffic Engineering methodology
The methodology is depicted on Figure 4. It involves the following steps:
$\rightarrow \quad$ Step 1) First optimize in an off-line procedure the routing pattern according to the performance criteria chosen (for example, try to minimize the load of the heaviest loaded link) allowing either all sub-optimality compliant single-path routing patterns or unique shortest paths routing patterns only. The output is a single-path routing pattern satisfying the sub-optimality condition;
$\rightarrow \quad$ Step 2) Search a metric compatible with a number of paths in this routing pattern equal to the number of compatible paths of the routing pattern. This step can also include some extra constraints provided that they can be expressed using a linear formulation (for example, equalities or inequalities verified by the metric values, minimizing the value changes from an existing metric set);
$\rightarrow \quad$ Step 3) If the metric obtained in Step 2) is not compatible with the entire routing pattern obtained in Step 1), create the necessary MPLS tunnels (ER-LSP) in order to reproduce completely the routing pattern obtained in Step 1) (Section 5.3.2);
$\rightarrow \quad$ Step 4) Then try to improve the routing performance of the solution obtained in Step 3) by adding a few MPLS tunnels: it is necessary in this step to find a tradeoff between the number of tunnels created and the gain in performance.

We can identify two different parts in this methodology. The first one (Steps 1 through 3) implies the modification of administrative metric values of the IGP in the network. This operation is not desirable too often. This type of action can be considered in a medium or long term basis. The second part of the methodology only attempts to create (or modify) MPLS tunnels in order to improve the routing performance. The tunnel creation and the resulting modification of the routing pattern (calculated by the modified IGP) are simple and fast operations (compared to the IGP convergence). This can be considered as a short term action.

One of the advantages of this TE methodology is to rely as much as possible on the IGP routing which has already proven its scalability, reliability and which is automated. The administrative metric values are changed when needed in order to optimize the routing performance of the nominal routing pattern. The use of MPLS tunnels enables the network operator to significantly improve the routing
performance in response to events in the network (transient change of traffic profile etc.) while limiting the number of MPLS tunnels which limits the complexity of management.

### 6.2 An ECMP based off-line Traffic Engineering methodology

We assume that the routers are able to split the traffic through different equal cost paths (see Section 3.2). The load splitting parameters have to be administratively configured.

The methodology involves the following steps:
$\rightarrow \quad$ Step 1) First compute off-line a multi-path routing pattern optimizing the performance criteria chosen (for example, try to minimize the load of the heaviest loaded link). This is generally easy to achieve (see Section 7.2);
$\rightarrow \quad$ Step 2) Determine the destination based multi-path routing pattern that achieves the same load links. In other words, determine the adequate load balancing parameters at each intermediate node and for each destination so that the resulting hop-by-hop routing achieves the same link loads (see Section3.2);
$\rightarrow \quad$ Step 3) Compute a metric compatible with this routing pattern (see Section 7.1.3).

We note that with this methodology, both IGP metrics and load balancing parameters must be administratively configured. The operation of modification of administrative metric values of the IGP in the network can be considered in a medium or long term basis. The operation of modifying load balancing parameters however does not have any convergence consequence. This could be done on a more frequent basis in response to events in the network (transient change of traffic profile etc.).

## 7 Algorithms for traffic engineering

In this Section we briefly present some of the algorithms used to address the problems that arise in the context of traffic engineering as described above. Due to space limitation, it is not possible to give in this paper either the proofs or the whole details of the algorithms. However, this Section is selfcontained and can be understood easily.

### 7.1 Compatible metrics

This Section is devoted to methods used to compute a set of edge metrics compatible with a set of routing paths.

### 7.1.1 Unique Shortest Paths

First let us focus on the case of unique shortest paths. As said in Section 3, the sub-optimality condition (Figure 1) of the routing paths is a necessary condition to find a set of compatible metrics.

Let $G=(V, E)$ be the graph associated with the network. The set of node pairs of $G$ for which a routing path R is given is denoted by K . In other terms, we assume that a path $\mathrm{R}(\mathrm{a}, \mathrm{b})$ is given for each $(a, b) \in K$. If c and d are such that $c \in R(a, b)$ and $d \in R(c, b)$, then $\mathrm{R}(\mathrm{c}, \mathrm{d})$ is assumed to be the subpath of $R(a, b)$ linking $c$ to $d$ (by sub-optimality). $S(a, b)$ is defined as the set of paths between a and $b$ which are different to $\mathrm{R}(\mathrm{a}, \mathrm{b})$. The metric is denoted by $\left(m_{e}\right)_{e \in E}$.

A general linear model that can be used to find metrics is the following:

$$
\left(L P_{1}\right)\left\{\begin{array}{l}
\text { Find }\left(m_{e}\right)_{e \in E} \\
\text { Subject to: } \\
\sum_{e \in R(a, b)} m_{e}=y_{a b} ; \forall(a, b) \in K \\
\sum_{e \in p} m_{e} \geq 1+y_{a b} ; \forall(a, b) \in K, p \in S(a, b) \\
m_{e} \geq 0 ; \forall e \in E
\end{array}\right.
$$

This linear program can be solved by generalized linear programming. An equivalent polynomial formulation can also be given [Ben-Ameur\&Gourdin_1][Ben-Ameur\&Liau]. If a solution is found, the metric given by $L P_{1}$ is compatible with the routing paths: every path $R(a, b)$ is a unique shortest path, according to this metric, between a and b .

Note that many particular constraints can be added to $\mathrm{LP}_{1}$ :

- All the metric values must be larger than 1;
- We may also want some links to have equal metrics;
- The routing paths used during failures are also given in advance (they must be shortest paths in the resulting graph obtained after the failure);
- The metrics may be required to be integer.

LP1 can also be solved considering various kinds of objective functions: minimize the maximum metric, the sum of metrics, or any linear function of the variables etc.

Note that $\mathrm{LP}_{1}$ does not always have a solution. Said another way, the sub-optimality condition is a necessary but not always a sufficient condition to find a metric. Some other necessary conditions are proposed in [Ben-Ameur\&Gourdin_1]. However, we showed that the sub-optimality is sufficient for some graphs such as cycles, cactus etc.

In the case where there is no feasible solution, an interesting particular formulation of $\mathrm{LP}_{1}$ is the one maximizing the number of demands whose routing paths are unique shortest paths (or equivalently that maximizes the number of compatible paths):

$$
\left(L P_{2}\right)\left\{\begin{array}{l}
\text { Maximize } \sum_{(\mathrm{a}, \mathrm{~b}) \in \mathrm{K}} \varepsilon_{a b} \\
\text { Subject to: } \\
\sum_{e \in R(a, b)} m_{e}=y_{a b} ; \forall(a, b) \in K \\
\sum_{e \in p} m_{e} \geq \varepsilon_{a b}+y_{a b} ; \forall(a, b) \in K, p \in S(a, b) \\
m_{e} \geq 0 ; \forall e \in E \\
0 \leq \varepsilon_{a b} \leq 1 ; \forall(a, b) \in K
\end{array}\right.
$$

$\mathrm{LP}_{2}$ always has a solution. It is also easy to show that the variables $\boldsymbol{\varepsilon}_{a b}$, obtained by solving $\mathrm{LP}_{2}$, will be equal to 1 and 0 . Said another way, $\mathrm{LP}_{2}$ gives exactly the demands that can be satisfied (in terms of unique shortest path constraint). The objective function of $\mathrm{LP}_{2}$ can also be more general.

### 7.1.2 Single-path routing with metrics and tunnels

When a compatible metric cannot be found (because the routing pattern is not compatible or because extra constraints have been added to the linear program), the routing pattern can be reproduced by introducing a few tunnels in order to modify the IGP routing according to the "IGP Shortcut" model (Section 5.3.2). In order to minimize the number of MPLS tunnels that need to be added a linear formulation slightly different from $\mathrm{LP}_{2}$ can be used. Instead of considering all the paths of $S(\mathrm{a}, \mathrm{b})$, we consider only the set $N(a, b)$ of paths that are node disjoint with $R(a, b)$. The program solved is the following.

$$
\left(M I P_{3}\right)\left\{\begin{array}{l}
\text { Minimize the numberoftunnels }=\sum_{(\mathrm{a}, \mathrm{~b}) \in \mathrm{K}} t_{a b} \\
\text { Subject to: } \\
\sum_{e \in R(a, b)} m_{e}=y_{a b} ; \forall(a, b) \in K \\
\sum_{e \in p} m_{e} \geq 1-\varepsilon_{a b}+y_{a b} ; \forall(a, b) \in K, p \in N(a, b) \\
0 \leq m_{e} \leq M ; \forall e \in E \\
\varepsilon_{a b} \geq 0 ; \forall(a, b) \in K \\
t_{a b} \geq \frac{\varepsilon_{a b}}{1+\|R(a, b)\| M} ; \forall(a, b) \in K \\
t_{a b} \in\{0,1\} ; \forall(a, b) \in K
\end{array}\right.
$$

We assume in $\mathrm{MIP}_{3}$ that the metric values are bounded by a maximum value M . We also use $\|\mathrm{R}(\mathrm{a}, \mathrm{b})\|$ to denote the number of hop of route $R(a, b)$. The variable $t_{a b}$ indicates if it is necessary to create a tunnel between $a$ and $b$. Note that a tunnel is created only if there is a path disjoint with $R(a, b)$ having a cost less or equal to the cost of $R(a, b)$. In the other cases, even if $R(a, b)$ is not a unique shortest path, we do not need a tunnel between a and b because some other intermediate tunnels will be created and used by the demand ( $a, b$ ) ("IGP Shortcut" model).

MIP $_{3}$ can be replaced by other easier linear programs that give a good approximation of the number of tunnels (without the upper bound M ):

$$
\left(L P_{4}\right)\left\{\begin{array}{l}
\text { Minimize } \sum_{(\mathrm{a}, \mathrm{~b}) \in \mathrm{K}} \varepsilon_{a b} \\
\text { Subject to: } \\
\sum_{e \in R(a, b)} m_{e}=y_{a b} ; \forall(a, b) \in K \\
\sum_{e \in p} m_{e} \geq 1-\varepsilon_{a b}+y_{a b} ; \forall(a, b) \in K, p \in N(a, b) \\
0 \leq m_{e} ; \forall e \in E \geq 0 \\
0 \leq \varepsilon_{a b} \leq 1 ; \forall(a, b) \in K
\end{array}\right.
$$

### 7.1.3 Multi-path routing pattern

We assume that a set of paths $R_{1}(a, b), R_{2}(a, b), \ldots$, is given between each pair of vertices $(a, b) \in K$. We would like to compute a metric such that all these paths are shortest paths. Let $\mathrm{C}(\mathrm{a}, \mathrm{b})$ be the set of paths between $a$ and $b$ different from the given routing paths $R_{1}(a, b), R_{2}(a, b), \ldots$.

Obviously, a null metric is a solution of the problem. However, for practical reasons, we want to minimize the number of links with a null metric value. This is formulated below:

$$
\left(L P_{5}\right)\left\{\begin{array}{l}
\text { Minimize } \sum_{\mathrm{e} \in \mathrm{E}} \varepsilon_{e} \\
\text { Subject to: } \\
\sum_{e \in R_{i}(a, b)} m_{e}=y_{a b} ; \forall(a, b) \in K, \forall R_{i}(a, b) \\
\sum_{e \in p} m_{e} \geq y_{a b} ; \forall(a, b) \in K, \forall p \in C(a, b) \\
m_{e} \geq 1-\varepsilon_{e} ; \forall e \in E \\
0 \leq \varepsilon_{e} \leq 1 ; \forall e \in E
\end{array}\right.
$$

The optimal solution of $\mathrm{LP}_{5}$ is necessarily integer: variables $\varepsilon_{e}$ will be equal to 0 or 1 .
Recall that any optimal multi-path routing without particular routing constaints (such as length constraints), can be seen as an optimal routing based only on destination. As $\mathrm{LP}_{5}$ provides a metric which is compatible with any multi-path routing, we can deduce that it is possible to optimize the network performance only by using a modified ECMP mechanism (Section 3.2). Said another way, first we have to compute an optimal multiflow optimizing the performance criterion (for example the maximum load). Then, we can determine the load balance coefficients by very simple calculations and transform the multiflow into a multi-path routing based only on destinations. Finally, we compute the edge metrics solving $\mathrm{LP}_{5}$ (or any other variation of $\mathrm{LP}_{5}$ ).

### 7.2 Optimization algorithms

Routing performance optimization is often a non trivial problem. Adequate models and methods have to be developed to address each specific problem. Often an exact resolution will not be possible in a reasonable computational time because some problems are NP-hard. In such cases efficient heuristics have to be found. Note that the difficulty of the optimization problem associated with a given routing strategy can be a decision criterion for an operational application. We present briefly in this Section the different problems and how they can be addressed.

### 7.2.1 Multi-path routing strategies

When multi-path routing is considered, the problem may be easy to solve. For example, if the optimization criterion is the maximum load or any linear function depending on edge loads, then the problem is polynomial (classical multiflow problem). Moreover, it is easy to integrate some additional constraints. For example on can restrict the problem to paths with limited number hops etc.

Multiflow problems are very classical. However, some simple and important results are not very known. Suppose for example that we would like to minimize the maximum load. It is very easy o show that we can find an optimal solution such that the number of used paths is lower than the number of
demands plus the number of edges. This means that many demands in an optimal solution will be singlepath routed.

### 7.2.2 Single-path routing strategies

For general single-path optimization problems, we use the tool described in [Geffard]. This tool is based on a branch\&cut algorithm.

The single-path routing with sub-optimality condition was studied in [Ben-Ameur\&Gourdin_2]. The algorithm used to compute a metric satisfying the sub-optimality condition is based on a cutting plane algorithm. To impose the sub-optimality condition, we define two new sets of 0-1variables: $r_{E}{ }^{k}$ and $r_{v}{ }^{k}$ for each traffic demand $k$, each vertex $v$ and each edge $e$. The sub-optimality condition can be written in the following way:

$$
r_{e}^{a, b} \geq r_{c}^{a, b}+r_{e}^{a, c}-1
$$

Many valid inequalities have been introduced to accelerate the algorithm of [BenAmeur\&Gourdin_2].

Finally, the optimization problems corresponding to shortest path routing strategies have been solved using some local search algorithms (see [Ben-Ameur\&al] and [Michel\&al]). The advantages of this method are, first its flexibility: it can be used for different kinds of optimization criterion and can integrate various constraints related to quality of service. Second it can solve large size problems. The main principle of these algorithms consists in changing the metric of some edges and re-computing the routing paths at each iteration. Some survivability constraints and the multi-hour behavior of the traffic have been considered in [Ben-Ameur]. Other heuristics have been proposed in [Pioro\&al] and [Thorup\&al].

## 8 Conclusion

To summarize, we describe new intra-domain routing mechanisms in IP network and how they can improve routing flexibility and performance in IP networks. Based on some numerical results, we then propose two different off-line Traffic Engineering methodologies that illustrate two possible evolutions of IP routing in intra-domain networks. Necessary algorithms to implement those methodologies are also shortly presented.

## A) MPLS BASEd Traffic Engineering methodology

A new mechanism like MPLS tunnels explicit routing gives more control over routing in IP networks. Various routing strategies for best effort traffic using this new functionality can be considered and all possible routing patterns can be realized in IP intra domain network. These routing strategies give more or less flexible control over the routing of the traffic but should also be compared in terms of complexity, scalability and robustness.

The comparison of the performance of these different routing strategies with the criteria of the heaviest loaded link shows that :

- The difference in terms of routing performance of the different routing strategy seems to strongly depend on the size and topology of the studied networks (which is not very surprising). It is thus important to focus on relevant topologies for IP networks;
- Whatever the routing strategy considered, optimization has an important consequence on the routing performance. This is specially true for the strategy of unique shortest path routing according to an administrative metric: a wise choice of the metric can significantly improve the routing performance;
- A routing strategy that permits to realize much more various routing patterns can not necessarily achieve a significantly better performance. A unique shortest path routing strategy performs very well in general and sometimes close to the optimum achievable with single-path or even multi-path routing strategies;
- The use of explicitly routed MPLS tunnels can improve the performance of routing. We show however that it is not necessary to rely only on explicit routing (which requires a large number of tunnels), but that mixed routing strategies based on IGP routing and MPLS tunnels can produce very interesting routing patterns in terms of performance. We give an algorithm minimizing the number of MPLS tunnels that need to be added to reproduce a given single-path routing pattern;

Based on those results, an off-line Traffic Engineering methodology is proposed. It is based on an optimization of the IGP routing (by a wise choice of the administrative metrics) enhanced by the use of a limited number of explicitly routed MPLS tunnels. Advantages of such a Traffic Engineering system would be to benefit from the highly proven robustness of the IGP routing while improving the performance and reactivity of the routing control in terms of resource utilization with a limited added operational complexity.

## B) ECMP BASED TrafFIC ENGINEERING METHODOLOGY

We assume that routers are able to split the traffic towards one destination on multiple paths according to some administratively defined load balancing parameters. It is then possible to reproduce the same (optimal) link loads in the network as those resulting from any given (optimal) multi-path routing pattern. This does not require any MPLS tunnels.

However MPLS can integrate various types of routing constraints allowing to implement specific routing strategies and QoS policies.

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[^0]:    ${ }^{1}$ Results for the Villamizar scenarios are directly reported from his Web site [Villamizar_2].

